**Probability practice**

**Part A.** Visitors to your website are asked to answer a single survey question before they get access to the content on the page. Among all of the users, there are two categories: Random Clicker (RC), and Truthful Clicker (TC). There are two possible answers to the survey: yes and no. Random clickers would click either one with equal probability. You are also giving the information that the expected fraction of random clickers is 0.3. After a trial period, you get the following survey results: 65% said Yes and 35% said No. What fraction of people who are truthful clickers answered yes? Hint: use the rule of total probability.

**Part B.** Imagine a medical test for a disease with the following two attributes:

* The sensitivity is about 0.993. That is, if someone has the disease, there is a probability of 0.993 that they will test positive.
* The specificity is about 0.9999. This means that if someone doesn't have the disease, there is probability of 0.9999 that they will test negative.
* In the general population, incidence of the disease is reasonably rare: about 0.0025% of all people have it (or 0.000025 as a decimal probability).

Suppose someone tests positive. What is the probability that they have the disease?

**Solution –**

**Part A:**

Given:

• The total fraction of Random Clickers (RC) is 0.3.

• The total fraction of Truthful Clickers (TC) will then be 1−0.3=0.7, since there are only these two categories.

• Random Clickers will click "Yes" 50% of the time and "No" 50% of the time since they are clicking randomly.

• The overall fraction of users who clicked "Yes" is 65% (0.65), and the fraction who clicked "No" is 35% (0.35).

Solution:

Using the law of total probability, we can write the fraction of "Yes" answers as a weighted sum of the fractions of "Yes" answers from the two groups (RC and TC):

0.65=fraction of RC who said Yes × probability of RC + fraction of TC who said Yes × probability of TC

Since the fraction of RC who said Yes is 0.5, and the probabilities of RC and TC are 0.3 and 0.7 respectively, we can substitute these values into the equation:

0.65=0.5×0.3+fraction of TC who said Yes×0.7

Simplifying, we have:

0.65=0.15+fraction of TC who said Yes×0.7

Subtracting 0.15 from both sides, we get:

0.5=fraction of TC who said Yes×0.7

Finally, dividing by 0.7, we find:

fraction of TC who said Yes=0.5/0.7≈**0.7143**

So, the fraction of Truthful Clickers who answered "Yes" is approximately **71.43%.**

**Part B:**

We are tasked with finding out the probability that someone actually has the disease given that they tested positive. This probability can be determined using Bayes' theorem.

Let's define:

• D: The event that someone has the disease.

• +: The event that someone tests positive.

We aim to calculate: P(D∣+) Which represents the probability of having the disease given a positive test result.

According to Bayes' theorem: P(D∣+) = P(+∣D) ×P(D)/ P (+)

Where:

• P(+∣D) is the probability of testing positive given that one has the disease, also known as the sensitivity. This is given as 0.993.

• P(D) is the probability that someone has the disease, which is provided as 0.000025.

We also need to determine P (+), which is the total probability that someone tests positive. This is computed using the total law of probability: P (+) =P(+∣D) ×P(D)+P(+∣notD) ×P(notD)

Where:

• P(+∣notD) is the probability of testing positive given that one doesn't have the disease. This is the complement of the specificity, or 1−0.9999=0.00011−0.9999=0.0001.

• P(notD) is the probability that someone does not have the disease. This is 1−0.000025=0.9999751−0.000025=0.999975.

Plugging in our known values:

P (+) =0.993×0.000025+0.0001×0.999975

P (+) =0.000024825+0.0000999975=0.0001248225

Now, using our Bayes' theorem formula:

P(D∣+) =0.993×0.000025/0.0001248225

P(D∣+) ≈**0.19888241**

In conclusion, given that someone tests positive for the disease, there's an approximate probability of **19.88%** that they genuinely have the illness.